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# Unsteady Laminar Film Condensation on Vertical Plate

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AIR FORCE SYSTEMS COMMAND
UNITED STATES AIR FORCE
*Inglewood, California*

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# UNSTEADY LAMINAR FILM CONDENSATION ON VERTICAL PLATE

by

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## ABSTRACT

The heat transfer associated with unsteady film condensation is analyzed for a vertical plate. The unsteady state is considered to be created by the time-dependent variation of either the uniform wall temperature or the g-force field. From the perturbation-type theoretical treatment, a set of universal variables and functions is derived which describes the unsteady behavior of the film. The universal functions are evaluated and tabulated so that the deviation of the heat transfer from the instantaneous steady state value can be computed readily. The effect of varying the fluid properties on the unsteady heat transfer is discussed.

CONTENTS

## SYMBOLS*

| | |
|---|---|
| $C$ | $(g/4\nu^2)^{1/4}$ |
| $c_p$ | Specific heat at constant pressure |
| $c_{pv}$ | Specific heat of vapor at constant pressure |
| $F$ | Steady-state nondimensional stream function |
| $f$ | Unsteady-state nondimensional stream function defined by Equation (14) |
| $f_0, f_1, \ldots f_{00}$ | Perturbed nondimensional stream functions |
| $g$ | Acceleration body force |
| $H$ | Steady-state nondimensional temperature |
| $h^o$ | Heat of condensation of vapor |
| $k$ | Thermal conductivity |
| $n$ | Positive integer |
| $Pr$ | Prandtl number |
| $q$ | Local heat transfer to wall |
| $T$ | Absolute temperature |
| $T_v$ | Absolute temperature of vapor |
| $t$ | Time |
| $u$ | The x-component of velocity |
| $v$ | The y-component of velocity |
| $x$ | Direction and distance along plate measured from leading edge |
| $y$ | Direction and distance normal to plate measured from wall |

*Note: All symbols are for liquid film unless specified otherwise.

| | |
|---|---|
| $\Gamma$ | Steady-state nondimensional film thickness |
| $\Delta$ | Unsteady-state nondimensional film thickness defined by Equation (12) |
| $\Delta_0, \Delta_1, \ldots \Delta_{00}$ | Perturbed nondimensional film thickness |
| $\delta$ | Film thickness |
| $\eta$ | Nondimensional similarity variable defined by Equation (10) |
| $\theta$ | Unsteady-state nondimensional temperature defined by Equation (15) |
| $\theta_0, \theta_1, \ldots \theta_{00}$ | Perturbed nondimensional temperature |
| $\nu$ | Kinematic viscosity |
| $\{\xi_n\}$ | Infinite set of nondimensional variables defined by Equations (19) and (20) |
| $\rho$ | Density |
| $\psi$ | Stream function defined by Equations (13) |

<u>Superscript</u>

| | |
|---|---|
| ' | Total differentiation with respect to the variable concerned |

<u>Subscript</u>

| | |
|---|---|
| s | Liquid-vapor interface |
| st | Instantaneous steady state value |
| w | Wall |

## INTRODUCTION

Film condensation is one of the basic heat transfer processes. The steady state problem has been widely studied since the time of Nusselt.[1]*

Sparrow and Siegel[2] analysed the relaxation period of the condensation process following a sudden drop of the wall temperature below the condensation temperature. The analysis was based on an approximate method which neglects the convection terms in the governing equations and assumes a linear temperature profile across the liquid film.

Often, in engineering applications, the condensation takes place under conditions which are continuously unsteady. The unsteady situation is usually caused by a time-dependent variation of the wall temperature. With the current interest in space technology, a factor in addition to unsteady wall temperature may become important to unsteady condensation--that is the unsteady force field which prevails in a space vehicle during acceleration and deceleration, and which also prevails during reduced-g experiments.

In the present paper, the complete boundary layer equations will be used to study the unsteady film condensation process and accompanying heat transfer for a plate located parallel to the acceleration field. The analysis applies when either the uniform wall temperature or the acceleration field is arbitrarily time dependent.

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*Superscript numbers in parentheses refer to similarly numbered references in bibliography at end of paper.

## FORMULATION OF PROBLEM

The physical model studied is shown on Fig. 1. The plate is in contact with uniform saturated vapor*, and the plate temperature is considered to be always below the saturation temperature. The vapor, therefore, continuously condenses at the liquid-vapor interface. The heat of condensation is transferred to the plate across the condensate film which flows along the plate under the influence of the body force. The process is unsteady here because either the wall temperature $T_w$ or the body force $g$ is arbitrarily time dependent. In the present paper, we are interested in studying the unsteady behavior of the liquid film so that the heat transfer to the plate may be obtained.

It is assumed, in the usual manner, that the condensation process is controlled by the flow within the liquid layer and is not limited by the supply of vapor at the interface. That is, the vapor reacts instantaneously to the inflow requirements defined by the liquid flow conditions at the liquid-vapor interface.

The following boundary-layer-type conservation equations describe the behavior of the liquid film. The density, viscosity, and the Prandtl number of the liquid are assumed to be constant and only the case of laminar liquid flow is considered.

Continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

Momentum equation:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g \tag{2}$$

---

*The analysis is also applicable when the vapor is superheated provided that $c_{pv}(T_v - T_s) \ll h^o$.

Energy equation:

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{\nu}{Pr}\ \frac{\partial^2 T}{\partial y^2} \tag{3}$$

The boundary conditions are as follows:

For y = 0

$$u = 0 \tag{4}$$

$$v = 0 \tag{5}$$

$$T = T_w(t) \tag{6}$$

for $y = \delta(x, t)$

$$\frac{\partial u}{\partial y} = 0 \tag{7}$$

$$T = T_s \tag{8}$$

$$\rho\left(\frac{\partial \delta}{\partial t} + u\frac{\partial \delta}{\partial x} - v\right)h^o = k\frac{\partial T}{\partial y} \tag{9}$$

Boundary condition (7) states that the shear at the liquid-gas interface is negligible. A recent study of steady-state condensation (3) which included the interface shear showed that the boundary condition (7) is sufficiently accurate for most cases: the only exception being the case where the Prandtl number is extremely small and, at the same time, the parameter $\frac{c_p(T_s - T_w)}{h^o}$ is large.

Boundary condition (9) is derived from the consideration that the heat released at the interface by the condensing vapor is carried away into the film by conduction. The momentum equation is coupled to the energy equation only through the boundary condition (9).

## SOLUTION OF EQUATIONS

In the present section, we shall solve the governing equations for the velocity and temperature profiles through the film, and for the film thickness $\delta$. The results will then be used to obtain the heat transfer.

A study of the transient analysis given by Sparrow and Siegel[2] suggests that the response of the film to the time-dependent variation of the boundary condition may be very fast. Therefore, we shall obtain an unsteady solution in the form of perturbation to the instantaneous steady-state solution. Such a solution would be most useful in determining when the heat transfer with either time-dependent wall temperature or $g$ can be computed with sufficient accuracy from quasisteady relations. The author[4] analyzed the problem of unsteady free convection in this manner. The governing equations, Equations (1) through (3), are quite similar to those used in the free convection study of reference (4). The basic differences between the analysis of reference (4) and the present analysis lie in the boundary conditions and, particularly, in the positions at which they must be applied. In the analysis of the unsteady free convection, the boundary conditions are applied at $y = 0$ and $y = \infty$. In the present analysis, the boundary conditions must be applied at $y = 0$ and $y = \delta$. The film thickness $\delta$ is not known a priori, and it may be obtained only as a result of the solution. The thickness, moreover, varies along the plate and with respect to time. In order to handle this complexity and also to arrive at the particular form of solutions sought, Equations (1) through (3) and their boundary conditions are first transformed in the following manner.

We may define a set of dimensionless variables as

$$\eta = \frac{y}{\delta(x, t)} = \frac{C}{x^{1/4}} \frac{y}{\Delta(\xi_0, \xi_1, \cdots \xi_n, \cdots)} \tag{10}$$

and

$$\left\{\xi_n(x, t)\right\} = \xi_0(x, t), \ \xi_1(x, t), \ \ldots\ldots \xi_n(x, t), \ \ldots \tag{11}$$

where

$$C = \left[g(t)/4\nu^2\right]^{1/4} .$$

Equation (10) defines the ordinate $\eta$ in such a way that the boundary conditions on all the subsequent transformed equations may be applied at $\eta = 0$ and $\eta = 1$ only. It can be seen from Equation (10) that the variable $\Delta$ represents a nondimensionalized liquid film thickness and is

$$\Delta(\xi_0, \xi_1, \ldots \xi_n, \ldots) = \delta(x, t) \left[g(t)/4\nu^2 x\right]^{1/4} . \tag{12}$$

For the time being, we let the set of variables $\left\{\xi_n(x, t)\right\}$ be a set of arbitrary functions of x and t.

The continuity equation (1) is satisfied in the usual manner by defining a stream function $\psi(x, y, t)$ by the equations:

$$\left.\begin{aligned} u &= \frac{\partial\psi}{\partial y} , \\ v &= -\frac{\partial\psi}{\partial x} . \end{aligned}\right\} \tag{13}$$

We now introduce a nondimensional stream function f and let it be related to $\psi$ by the following equation:

$$f(\eta, \xi_0, \xi_1, \ldots \xi_n, \ldots) = \frac{\psi(x, y, t)}{4\nu x^{3/4} C(t)\Delta(\xi_0, \xi_1, \ldots \xi_n, \ldots)} . \tag{14}$$

We also define a nondimensional temperature function as:

$$\theta(\eta, \xi_0, \xi_1, \cdots \xi_n, \cdots) = \frac{T(x, y, t) - T_s}{T_w(t) - T_s} \quad . \tag{15}$$

In Equations (10), (12), (14), and (15), it was assumed a priori that $\Delta$ is a function of $\left\{\xi_n(x, t)\right\}$, and f and $\theta$ are functions of $\eta$ and $\left\{\xi_n(x, t)\right\}$ only. Now if the transformations

$$\delta(x, t) \longrightarrow \Delta(\xi_0, \xi_1, \cdots \xi_n, \cdots)$$

$$\psi(x, y, t) \longrightarrow f(\eta, \xi_0, \xi_1, \cdots \xi_n, \cdots)$$

and

$$\theta(x, y, t) \longrightarrow \theta(\eta, \xi_0, \xi_1, \cdots \xi_n, \cdots)$$

are carried out in Equations (2) through (9), and if the resulting equations could be made to be functions of $\eta$ and $\left\{\xi_n(x, t)\right\}$ only, then the a priori assumptions would have been proved to be self-consistent ones for the present purpose.

The definitions of nondimensional stream function and the variables yield:

$$u = 4\nu C^2 x^{1/2} \frac{\partial f}{\partial \eta}$$

and

$$\left.\begin{aligned} v = -\nu C x^{-1/4} \Delta\left(3f - \eta\frac{\partial f}{\partial \eta}\right) - 4\nu C x^{3/4}\left(f - \eta\frac{\partial f}{\partial \eta}\right)\sum_{n=0}^{\infty}\frac{\partial \Delta}{\partial \xi_n}\frac{\partial \xi_n}{\partial x} \\ - 4\nu C x^{3/4}\Delta\sum_{n=0}^{\infty}\frac{\partial f}{\partial \xi_n}\frac{\partial \xi_n}{\partial x} \quad . \end{aligned}\right\} \tag{16}$$

The momentum and the energy equations are now transformed to the following equations.

Momentum equation:

$$
\frac{\partial^3 f}{\partial \eta^3} + \Delta^2 \left[ 3f \frac{\partial^2 f}{\partial \eta^2} - 2\left(\frac{\partial f}{\partial \eta}\right)^2 + 1 \right] = \frac{C'}{C^3} \frac{x^{1/2}}{\nu} \Delta^2 \left( 2 \frac{\partial f}{\partial \eta} + \eta \frac{\partial^2 f}{\partial \eta^2} \right)
$$

$$
+ \frac{x^{1/2}}{\nu C^2} \left( \Delta^2 \sum_{n=0}^{\infty} \frac{\partial^2 f}{\partial \eta \partial \xi_n} \frac{\partial \xi_n}{\partial t} - \Delta \eta \frac{\partial^2 f}{\partial \eta^2} \sum_{n=0}^{\infty} \frac{\partial \Delta}{\partial \xi_n} \frac{\partial \xi_n}{\partial t} \right) + 4x\Delta^2 \left( \frac{\partial f}{\partial \eta} \sum_{n=0}^{\infty} \frac{\partial^2 f}{\partial \eta \partial \xi_n} \frac{\partial \xi_n}{\partial x} \right.
$$

$$
\left. - \frac{\partial^2 f}{\partial \eta^2} \sum_{n=0}^{\infty} \frac{\partial f}{\partial \xi_n} \frac{\partial \xi_n}{\partial x} \right) - 4x\Delta f \frac{\partial^2 f}{\partial \eta^2} \sum_{n=0}^{\infty} \frac{\partial \Delta}{\partial \xi_n} \frac{\partial \xi_n}{\partial x} \tag{17}
$$

Energy equation

$$\frac{1}{Pr}\frac{\partial^2\theta}{\partial\eta^2} + 3\Delta^2 f\frac{\partial\theta}{\partial\eta} = \frac{C'}{C^3}\frac{x^{1/2}}{\nu}\Delta^2\eta\frac{\partial\theta}{\partial\eta} + \frac{x^{1/2}}{\nu C^2}\left[\Delta^2\theta\frac{(T_w - T_s)'}{(T_w - T_s)}\right.$$

$$\left. -\Delta\eta\frac{\partial\theta}{\partial\eta}\sum_{n=0}^{\infty}\frac{\partial\Delta}{\partial\xi_n}\frac{\partial\xi_n}{\partial t} + \Delta^2\sum_{n=0}^{\infty}\frac{\partial\theta}{\partial\xi_n}\frac{\partial\xi_n}{\partial t}\right] + 4x\left(\Delta^2\frac{\partial f}{\partial\eta}\sum_{n=0}^{\infty}\frac{\partial\theta}{\partial\xi_n}\frac{\partial\xi_n}{\partial x}\right.$$

$$\left. -\Delta f\frac{\partial\theta}{\partial\eta}\sum_{n=0}^{\infty}\frac{\partial\Delta}{\partial\xi_n}\frac{\partial\xi_n}{\partial x} - \Delta^2\frac{\partial\theta}{\partial\eta}\sum_{n=0}^{\infty}\frac{\partial f}{\partial\xi_n}\frac{\partial\xi_n}{\partial x}\right) \qquad (18)$$

In the above two equations, $C'$ and $(T_w - T_s)'$ are zero when the body force and the wall temperature, respectively, are independent of time.

A study of Equations (17) and (18) shows that all the variables x and t appearing explicitly in these equations would disappear completely when $\xi_n$ is defined as:

$$\xi_n = -\left(\frac{x}{g}\right)^{\frac{n+1}{2}}\frac{1}{(T_s - T_w)}\frac{d^{n+1}(T_w - T_s)}{d\,t^{n+1}} \qquad (19)$$

when $T_w$ is unsteady, and

$$\xi_n = \left(\frac{x}{\nu^2}\right)^{\frac{n+1}{2}}\frac{1}{C^{3+2n}}\frac{d^{n+1}C}{d\,t^{n+1}} \qquad (20)$$

when g is unsteady. Here it is considered that $T_w$ and g are continuously differentiable with respect to t. The fact that the particular definitions of $\xi_n$ make equations (17) and (18) functions of $\eta$ and $\{\xi_n\}$ only, can be readily seen by noting the relationships;

$$\frac{x^{1/2}}{\nu C^2}\frac{\partial \xi_n}{\partial t} = 2\sqrt{\frac{x}{g}}\frac{\partial \xi_n}{\partial t} = \xi_{n+1} - \xi_0 \xi_n$$

$$x\frac{\partial \xi_n}{\partial x} = \left(\frac{n+1}{2}\right)\xi_n$$

when $T_w$ is unsteady, and

$$\frac{x^{1/2}}{\nu C^2}\frac{\partial \xi_n}{\partial t} = \xi_{n+1} - (3+2n)\xi_0 \xi_n$$

$$x\frac{\partial \xi_n}{\partial x} = \left(\frac{n+1}{2}\right)\xi_n \quad .$$

when g is unsteady.

Equations (17) and (18) now become for the case of unsteady $T_w$:

Momentum equation

$$\frac{\partial^3 f}{\partial \eta^3} + \Delta^2\left[3f\frac{\partial^2 f}{\partial \eta^2} - 2\left(\frac{\partial f}{\partial \eta}\right)^2 + 1\right] = 2\left[-\Delta\eta\frac{\partial^2 f}{\partial \eta^2}\sum_{n=0}^{\infty}\left(\xi_{n+1} - \xi_0\xi_n\right)\frac{\partial \Delta}{\partial \xi_n}\right.$$

$$+\Delta^2\sum_{n=0}^{\infty}(\xi_{n+1} - \xi_0\xi_n)\frac{\partial^2 f}{\partial \eta \partial \xi_n} + \Delta^2\frac{\partial f}{\partial \eta}\sum_{n=0}^{\infty}(n+1)\,\xi_n\frac{\partial^2 f}{\partial \eta \partial \xi_n}$$

$$\left. - \Delta f\frac{\partial^2 f}{\partial \eta^2}\sum_{n=0}^{\infty}(n+1)\,\xi_n\frac{\partial \Delta}{\partial \xi_n} - \Delta^2\frac{\partial^2 f}{\partial \eta^2}\sum_{n=0}^{\infty}(n+1)\,\xi_n\frac{\partial f}{\partial \xi_n}\right] \qquad (21)$$

Energy equation

$$\frac{1}{\Pr}\frac{\partial^2\theta}{\partial\eta^2} + 3\Delta^2 f\frac{\partial\theta}{\partial\eta} = 2\Bigg[-\Delta\eta\frac{\partial\theta}{\partial\eta}\sum_{n=0}^{\infty}(\xi_{n+1} - \xi_0\,\xi_n)\frac{\partial\Delta}{\partial\xi_n}$$

$$+ \Delta^2\sum_{n=0}^{\infty}(\xi_{n+1} - \xi_0\,\xi_n)\frac{\partial\theta}{\partial\xi_n} + \Delta^2\xi_0\theta + \Delta^2\frac{\partial f}{\partial\eta}\sum_{n=0}^{\infty}(n+1)\,\xi_n\frac{\partial\theta}{\partial\xi_n}$$

$$- \Delta f\frac{\partial\theta}{\partial\eta}\sum_{n=0}^{\infty}(n+1)\,\xi_n\frac{\partial\Delta}{\partial\xi_n} - \Delta^2\frac{\partial\theta}{\partial\eta}\sum_{n=0}^{\infty}(n+1)\,\xi_n\frac{\partial f}{\partial\xi_n}\Bigg] \tag{22}$$

Equations (17) and (18) become for the case of unsteady g:

Momentum equation

$$\frac{\partial^3 f}{\partial\eta^3} + \Delta^2\left[3f\frac{\partial^2 f}{\partial\eta^2} - 2\left(\frac{\partial f}{\partial\eta}\right)^2 + 1\right] = \xi_0\,\Delta^2\left(2\frac{\partial f}{\partial\eta} + \eta\frac{\partial^2 f}{\partial\eta^2}\right)$$

$$- \Delta\eta\frac{\partial^2 f}{\partial\eta^2}\sum_{n=0}^{\infty}\Big[\xi_{n+1} - (3+2n)\,\xi_0\,\xi_n\Big]\frac{\partial\Delta}{\partial\xi_n} + \Delta^2\sum_{n=0}^{\infty}\Big[\xi_{n+1} - (3+2n)\,\xi_0\,\xi_n\Big]\frac{\partial^2 f}{\partial\eta\partial\xi_n}$$

$$+ 2\Delta^2\frac{\partial f}{\partial\eta}\sum_{n=0}^{\infty}(n+1)\,\xi_n\frac{\partial^2 f}{\partial\eta\partial\xi_n} - 2f\frac{\partial^2 f}{\partial\eta^2}\Delta\sum_{n=0}^{\infty}(n+1)\,\xi_n\frac{\partial\Delta}{\partial\xi_n}$$

$$- 2\Delta^2\frac{\partial^2 f}{\partial\eta^2}\sum(n+1)\,\xi_n\frac{\partial f}{\partial\xi_n} \tag{23}$$

Energy equation

$$\frac{1}{Pr}\frac{\partial^2\theta}{\partial\eta^2} + 3\,\Delta^2 f\,\frac{\partial\theta}{\partial\eta} = \xi_0\,\Delta^2\,\eta\frac{\partial\theta}{\partial\eta} - \Delta\eta\,\frac{\partial\theta}{\partial\eta}\sum_{n=0}^{\infty}\left[\xi_{n+1} - (3+2n)\,\xi_0\,\xi_n\right]\frac{\partial\Delta}{\partial\xi_n}$$

$$+ \Delta^2\sum_{n=0}^{\infty}\left[\xi_{n+1} - (3+2n)\,\xi_0\,\xi_n\right]\frac{\partial\theta}{\partial\xi_n} + 2\,\Delta^2\,\frac{\partial f}{\partial\eta}\sum_{n=0}^{\infty}(n+1)\,\xi_n\,\frac{\partial\theta}{\partial\xi_n}$$

$$- 2\Delta f\,\frac{\partial\theta}{\partial\eta}\sum_{n=0}^{\infty}(n+1)\,\xi_n\,\frac{\partial\Delta}{\partial\xi_n} - 2\,\Delta^2\,\frac{\partial\theta}{\partial\eta}\sum_{n=0}^{\infty}(n+1)\,\xi_n\,\frac{\partial f}{\partial\xi_n} \tag{24}$$

The boundary conditions (4) through (8) are transformed for both unsteady -$T_w$ and -g cases as:

At $\eta = 0$

$$\theta = 1 \tag{25}$$

$$f = 0 \tag{26}$$

$$\partial f/\partial\eta = 0 \quad . \tag{27}$$

At $\eta = 1$

$$\theta = 0 \tag{28}$$

$$\partial^2 f/\partial\eta^2 = 0 \quad . \tag{29}$$

Boundary condition (9) is transformed at $\eta = 1$ for unsteady $-T_w$ case as:

$$2\Delta\sum_{n=0}^{\infty}(\xi_{n+1} - \xi_0\,\xi_n)\frac{\partial\Delta}{\partial\xi_n} + 3f\Delta^2 + 2\Delta f\sum_{n=0}^{\infty}(n+1)\,\xi_n\frac{\partial\Delta}{\partial\xi_n}$$

$$+ 2\Delta^2\sum_{n=0}^{\infty}(n+1)\,\xi_n\frac{\partial f}{\partial\xi_n} = -\frac{1}{Pr}\frac{c_p(T_s - T_w)}{h^o}\frac{\partial\theta}{\partial\eta} \tag{30}$$

and for unsteady -g case as:

$$-\Delta^2\,\xi_0 + \Delta\sum_{n=0}^{\infty}\left[\xi_{n+1} - (3+2n)\,\xi_0\,\xi_n\right]\frac{\partial\Delta}{\partial\xi_n} + 3\Delta^2 f + 2\Delta f\sum_{n=0}^{\infty}(n+1)\,\xi_n\frac{\partial\Delta}{\partial\xi_n}$$

$$+ 2\Delta^2\sum_{n=0}^{\infty}(n+1)\,\xi_n\frac{\partial f}{\partial\xi_n} = -\frac{1}{Pr}\frac{c_p(T_s - T_w)}{h^o}\frac{\partial\theta}{\partial\eta} \tag{31}$$

Equations (21) through (31) are functions of $\eta$ and $\left\{\xi_n(x,t)\right\}$ only. It is now clear that the initial assumptions made on $\Delta$, f, and $\theta$ are entirely self-consistent. That is, $\Delta$ is a function of $\left\{\xi_n(x,t)\right\}$, and f and $\theta$ are functions of $\eta$ and $\left\{\xi_n(x,t)\right\}$ only.

In order to solve Equations (21) through (24) with their boundary conditions, the functions f, $\theta$, and $\Delta$ are first expanded into generalized Taylor series about the steady-state solutions as follows:

$$f(\eta, \xi_0, \xi_1, \cdots \xi_n, \cdots) = F(\eta) + \left[\xi_0\,f_0(\eta) + \xi_1\,f_1(\eta) + \cdots\right]\cdots$$

$$+\left[\xi_0^2\,f_{00}(\eta) + \cdots + \xi_0\,\xi_1\,f_{01}(\eta) + \cdots\right]\cdots \tag{32}$$

$$\theta(\eta, \xi_0, \xi_1, \cdots \xi_n, \cdots) = H(\eta) + \left[\xi_0 \theta_0(\eta) + \xi_1 \theta_1(\eta) + \cdots\right]\cdots$$

$$+\left[\xi_0^2 \theta_{00}(\eta) + \cdots + \xi_0 \xi_1 \theta_{01}(\eta) + \cdots\right]\cdots \tag{33}$$

and

$$\Delta(\xi_0, \xi_1, \cdots \xi_n, \cdots) = \Gamma + \left[\xi_0 \Delta_0 + \xi_1 \Delta_1 + \cdots\right]\cdots$$

$$+\left[\xi_0^2 \Delta_{00} + \cdots + \xi_0 \xi_1 \Delta_{01} + \cdots\right]\cdots \tag{34}$$

**When the Series (32) through (34) are substituted into Equations (21) through (24) and when the coefficients of 1, $\xi_0$, $\xi_0^2$, $\xi_1$, etc.$\cdots$ are collected in each equation, an infinite set of perturbed equations results. For the case of unsteady $T_w$, the zeroth-order and the first two first-order perturbed equations are:**

$$\left.\begin{aligned} & F''' + \Gamma^2 (3FF'' - 2F'^2 + 1) = 0 \\ & \frac{1}{Pr} H'' + 3\Gamma^2 F H' = 0 \end{aligned}\right\} \tag{35}$$

$$\left.\begin{aligned} & f_0''' + \Gamma^2 (3Ff_0'' - 6F'f_0' + 5F''f_0) = -2\Gamma\Delta_0 (4FF'' - 2F'^2 + 1) \\ & \frac{1}{Pr} \theta_0'' + \Gamma^2 (3F\theta_0' - 2F'\theta_0) = 2\Gamma^2 H - 8\Gamma\Delta_0 FH' - 5\Gamma^2 f_0 H' \end{aligned}\right\} \tag{36}$$

$$
\left.
\begin{aligned}
& f_1''' + \Gamma^2(3Ff_1'' - 8F'f_1' + 7F''f_1) = -2\Gamma\Delta_1(5FF'' - 2F'^2 + 1) \\
& \qquad + 2(\Gamma^2 f_0' - \Gamma\Delta_0\eta F'') \\
& \frac{1}{Pr}\theta_1'' + \Gamma^2(3F\theta_1' - 4F'\theta_1) = -2\Gamma\Delta_0\eta H' + 2\Gamma^2\theta_0 \\
& \qquad - 10\Gamma\Delta_1 FH' - 7\Gamma^2 f_1 H'
\end{aligned}
\right\} \quad (37)
$$

For the case of unsteady g, the zeroth-order equations are the same as Equations (35) and the first two first-order perturbed equations are:

$$
\left.
\begin{aligned}
& f_0''' + \Gamma^2(3Ff_0'' - 6F'f_0' + 5F''f_0) = \Gamma^2(2F' + \eta F'') \\
& \qquad - 2\Gamma\Delta_0(4FF'' - 2F'^2 + 1) \\
& \frac{1}{Pr}\theta_0'' + \Gamma^2(3F\theta_0' - 2F'\theta_0) = \Gamma^2\eta H' - 8\Gamma\Delta_0 FH' - 5\Gamma^2 f_0 H'
\end{aligned}
\right\} \quad (38)
$$

$$
\left.
\begin{aligned}
& f_1''' + \Gamma^2(3Ff_1'' - 8F'f_1' + 7F''f_1) = -\Gamma\Delta_0\eta F'' + \Gamma^2 f_0' \\
& \qquad - 2\Gamma\Delta_1(5FF'' - 2F'^2 + 1) \\
& \frac{1}{Pr}\theta_1'' + \Gamma^2(3F\theta_1' - 4F'\theta_1) = -\Gamma\Delta_0\eta H' + \Gamma^2\theta_0 - 10\Gamma\Delta_1 FH' \\
& \qquad - 7\Gamma^2 f_1 H'
\end{aligned}
\right\} \quad (39)
$$

Boundary conditions (25) through (29) for both unsteady $-T_w$ and $-g$ cases become:

At $\eta = 0$

$$F = f_0 = f_1 = 0$$

$$F' = f_0' = f_1' = 0$$

$$H = 1, \ \theta_0 = \theta_1 = 0 \tag{40}$$

At $\eta = 1$

$$F'' = f_0'' = f_1'' = 0$$

$$H = \theta_0 = \theta_1 = 0 \tag{41}$$

Also at $\eta = 1$, boundary condition (30) becomes for unsteady $-T_w$ case

$$3\Gamma^2 F = -\frac{1}{Pr}\frac{c_p(T_s - T_w)}{h^o} H'$$

$$8\Gamma\Delta_0 F + 5\Gamma^2 f_0 = -\frac{1}{Pr}\frac{c_p(T_s - T_w)}{h^o}\theta_0'$$

$$2\Gamma\Delta_0 + 10\Gamma\Delta_1 F + 7\Gamma^2 f_1 = -\frac{1}{Pr}\frac{c_p(T_s - T_w)}{h^o}\theta_1' \tag{42}$$

and boundary condition (31) becomes for unsteady $-g$ case

$$3\Gamma^2 F = -\frac{1}{Pr}\frac{c_p(T_s-T_w)}{h^o} H'$$

$$-\Gamma^2 + 8\Gamma\Delta_0 F + 5\Gamma^2 f_0 = -\frac{1}{Pr}\frac{c_p(T_s-T_w)}{h^o}\theta_0'$$

$$\Gamma\Delta_0 + 10\Gamma\Delta_1 F + 7\Gamma^2 f_1 = -\frac{1}{Pr}\frac{c_p(T_s-T_w)}{h^o}\theta_1' \quad . \tag{43}$$

Equations (35) through (39) and their boundary conditions (40) through (43) show that the solution of each of the perturbed equations will depend on the parameters Pr and $c_p(T_s-T_w)/h^o$. Steady-state solutions of various condensation problems, when they are based on boundary layer equations, depend on the two parameters, Pr and $c_p(T_s-T_w)/h^o$, as is seen in references (3), (5), (6), and (7). With the particular transformations used in the present analysis, it is seen that we may evaluate the universal functions, F, $f_0$, H, $\theta_0$, $\Gamma$, $\Delta_0$, etc., as functions of the same two parameters only, and we may obtain the general unsteady solutions by the use of series (32) through (34).

Equations (35) are the equations for the steady-state case, and some of their solutions can be deduced from the results of reference (5) by properly relating the variables of the present analysis to those of the reference. Equations (35) through (39) are integrated in the present study by the use of IBM-7090 digital computer for several combinations of Pr and $c_p(T_s-T_w)/h^o$. Two typical results are shown in Figs. 2 through 5.

## HEAT TRANSFER

The heat transfer at the wall may be now obtained by evaluating $(\partial T/\partial y)_w$ from the results of the preceding section. The relationship between the instantaneous local heat transfer q and the hypothetical instantaneous steady-state transfer $q_{st}$ is derived by the use of Equations (32) through (34) as:

$$\frac{q}{q_{st}} = 1 + \xi_0 \left( \frac{\theta_{0w}'}{H_w'} - \frac{\Delta_0}{\Gamma} \right) + \xi_1 \left( \frac{\theta_{1w}'}{H_w'} - \frac{\Delta_1}{\Gamma} \right) + \cdots \tag{44}$$

where

$$q_{st} = k(T_w - T_s) \left( \frac{g}{4\nu^2 x} \right)^{1/4} \frac{\ddot{H}_w'}{\Gamma} \quad . \tag{45}$$

The computed values of $H'_w$, $\theta_{0w}'$, $\Delta_0$, etc., are tabulated in Tables 1 and 2. The coefficients of $\xi_0$ and $\xi_1$ of Equation (44) calculated from the tables are presented in a graphical form in Figs. 6 through 9. These figures and Equation (44) together enable us to quickly evaluate the ratio of unsteady local heat transfer to the hypothetical instantaneous steady-state heat transfer when $\xi_n$ are sufficiently small.

## DISCUSSION

Figures 6 through 9 show the parameters $(\theta_{0w}'/H_w' - \Delta_0/\Gamma)$ and $(\theta_{1w}'/H_w' - \Delta_1/\Gamma)$, which will be referred to hereafter as coefficients of $\xi_0$ and $\xi_1$, respectively, calculated from Tables 1 and 2 for unsteady $-T_w$ and $-g$ cases. As can be seen from Equation (44), these parameters directly represent the first-order effect of unsteady $-T_w$ and $-g$ on the condensation heat transfer. Behavior of these parameters, therefore, will be discussed in the present section.

### Unsteady $T_w$ Case

Let us first investigate the relationships between the coefficients of $\xi_0$ and $\xi_1$, and Prandtl numbers. Figures 6 and 7 show that the effect of unsteady $-T_w$ on heat transfer becomes greater as the Prandtl number is increased. The reason is rather obvious. A larger Prandtl number implies that either the kinematic viscosity is higher or the thermal diffusivity is lower, or it implies both. As is seen from Equation (12), a higher $\nu$ means, in general, a thicker film. Both increasing film thickness and decreasing thermal

diffusivity increase the time lag in thermal response of the film to the varying $T_w$. Therefore, the deviation of unsteady heat transfer from the instantaneous steady-state value becomes greater as Prandtl number is increased.

It is seen in Figs. 6 and 7 that $(\theta_{0w}'/H_w' - \Delta_0/\Gamma)$ is positive whereas $(\theta_{1w}'/H_w' - \Delta_1/\Gamma)$ is negative. Equation (19) shows that both $\xi_0$ and $\xi_1$ are negative when $T_w'$ and $T_w''$ are positive. Therefore, in view of Equation (44), the first-order effect of $T_w'$ on heat transfer is to decrease it from the instantaneous steady-state value whereas that of $T_w''$ is to increase it.

It is also seen in the figures that the coefficient of $\xi_0$ increases approximately with $\sqrt{Pr}$ whereas the magnitude of the coefficient of $\xi_1$ increases approximately with Pr. This means that only the effect of $T_w'$ is important when the Prandtl number is low, and the importance of the effect of $T_w''$ in relation to that of $T_w'$ increases quite rapidly as the Prandtl number is increased.

In Table 1 and Figs. 6 and 7, only the results for $Pr \geq 1$ are presented. The reason for this is that the coefficients of $\xi_0$ and $\xi_1$ for the lower Prandtl numbers were found to be sufficiently small so that the effect of unsteady $T_w$ on heat transfer is practically negligible when $Pr << 1$.

The relationships between the coefficients of $\xi_0$ and $\xi_1$, and the parameter $c_p(T_s-T_w)/h^o$ can be also seen from Figs. 6 and 7. It is seen that, for a given Prandtl number, the magnitudes of coefficients of $\xi_0$ and $\xi_1$ increase as the parameter $c_p(T_s-T_w)/h^o$ increases. The film is usually thicker when $c_p(T_s-T_w)/h^o$ is larger (see $\Gamma$'s in Table 1). The time lag, therefore, in the thermal response of the film to the varying $T_w$ becomes greater as $c_p(T_s-T_w)/h^o$ increases.

Equation (19) shows that the magnitude of $\xi_n$ increases with $1/(T_s-T_w)$. Figures 6 and 7, on the other hand, show that the magnitudes of coefficients of $\xi_0$ and $\xi_1$ increase approximately with $\sqrt{T_s-T_w}$ . Hence, it is seen from Equation (44) that the first-order deviation of heat transfer from the instantaneous steady-state value increases approximately with $1/\sqrt{T_s-T_w}$ for given

values of $T_w'$ and $T_w''$. The first-order effect, therefore, of the unsteady $T_w$ on heat transfer increases as $(T_s-T_w)$ is decreased for given values of $T_w'$ and $T_w''$. It is interesting to note, however, that this dependence of the unsteady effect on the energy-driving potential is much weaker than that found in reference (4) for the unsteady convection problem.

The unsteady convective heat transfer depended on $\{\xi_n\}$ only and

$$\xi_0 \sim \frac{T_w'}{(T_w-T_\infty)^{3/2}}$$

$$\xi_1 \sim \frac{T_w''}{(T_w-T_\infty)^2} - \frac{3}{4}\,\frac{(T_w')^2}{(T_w-T_\infty)^3}$$

where $T_\infty$ is the undisturbed gas temperature.

<u>Unsteady g Case</u>

The effect of unsteady g on condensation heat transfer is of a different nature compared with the effect of unsteady $-T_w$ discussed in the preceding section. In the latter case, the unsteadiness is created at one of the boundaries, and the disturbance propagated into the film. The unsteady g, on the other hand, effects the entire film simultaneously since g is uniform throughout the flow field. Moreover, the unsteady $-T_w$ affects the temperature profile directly and the flow field indirectly whereas the reverse is true for the unsteady -g case.

There are two separate factors which cause time lag in the flow response of the film to the varying g. They are the inertial and the viscous drags of the liquid in the film. Let us investigate the behavior of these two factors and, in turn, the unsteady behavior of the film with respect to Prandtl number and

$c_p(T_s-T_w)/h^o$. The time lag due to inertia is independent of Pr and $c_p(T_s-T_w)/h^o$ since it is independent of the total mass of the film. The time lag due to viscous drag, on the other hand, depends quite strongly on Pr and $c_p(T_s-T_w)/h^o$. At the higher Prandtl numbers, the viscous effect is felt by most of the film. As the Prandtl number is decreased, the viscous effect becomes confined to the portion of the film closest to the wall. This phenomenon can be seen, for instance, by comparing the steady-state velocity profiles given in Figs. 2 and 4 for Pr = 10 and Pr = 0.01, respectively. Thus, the time lag in the flow response of the film to the varying g at high Prandtl numbers (Pr $\gtrsim$ 10) is predominantly due to the viscous drag, whereas the time lag at low Prandtl numbers (Pr $\lesssim$ 0.1) is mainly due to the inertial drag. This fact is clearly seen in Fig. 8 where the coefficients of $\xi_0$ and $\xi_1$ are plotted against Prandtl number for $c_p(T_s-T_w)/h^o$ = 0.1 . It is seen from Fig. 8 that the magnitudes of coefficients of $\xi_0$ and $\xi_1$ increase as the Prandtl number is increased when the Prandtl number is high. It is because the viscous drag continuously increases as the Prandtl number is increased. At the low Prandtl numbers, on the other hand, it is seen that the coefficients of $\xi_0$ and $\xi_1$ become practically invariant with respect to the Prandtl number since the inertial drag is independent of the Prandtl number. The transition from the regime of dominant inertial lag to the regime of dominant viscous lag is seen from Fig. 8 to take place between Pr $\approx$ 0.5 and Pr $\approx$ 5.

As mentioned earlier, the no-interface-shear boundary condition, Equation (7), becomes less accurate as the Prandtl number is decreased below about 0.1. There probably would be an additional time lag due to the interface shear, and this effect could become non-negligible at the low Prandtl numbers. The magnitudes of coefficients of $\xi_0$ and $\xi_1$ may, therefore, begin to increase again as the Prandtl number is continuously lowered below about 0.1.

Let us now consider the effect of varying $c_p(T_s-T_w)/h^o$ on the coefficients of $\xi_0$ and $\xi_1$. As previously noted, the film is generally thicker when $c_p(T_s-T_w)/h^o$ is larger for a given Prandtl number (as was seen from $\Gamma$'s given in Tables 1 and 2). When the film is thick, a relatively small portion of

the total liquid near the wall is strongly affected by viscous drag. On the other hand, when the film is thin, practically the entire film is affected strongly by the viscous drag. Therefore, with the aid of previous discussions, it is seen that the magnitudes of coefficients of $\xi_0$ and $\xi_1$ should increase as $c_p(T_s-T_w)/h^o$ is decreased. Figure 9, which shows the variations of coefficients of $\xi_0$ and $\xi_1$ with respect to $c_p(T_s-T_w)/h^o$, supports this argument for $Pr = 10$.

It is seen from Figs. 8 and 9 that the coefficient of $\xi_0$ is negative whereas that of $\xi_1$ is positive. At the same time, Equation (20) shows that $\xi_0$ and $\xi_1$ are positive when C' and C" are positive. Hence, the effect of C' on heat transfer is a decrease from the instantaneous steady-state value, whereas the effect of C" is an increase.

Finally, let us briefly consider the reduced-g experiments which are being performed by various people in connection with space applications. Conditions of unsteady g usually prevail during reduced-g experiments. It can be shown from Equation (20) that

$$\xi_0 \sim \frac{g'}{g^{3/2}} \quad \text{and} \quad \xi_1 \sim \left[ \frac{g''}{g^2} - \frac{3}{4} \frac{(g')^2}{g^3} \right] \quad .$$

Hence, the effect of unsteady g on condensation heat transfer becomes magnified quite rapidly with decreasing g for given values of g' and g". It is therefore important to keep the unsteady -g effect in mind when the reduced -g experiment includes condensation. The effect is most pronounced when the Prandtl number is high and, at the same time, $c_p(T_s - T_w)/h^o$ is low. It is noted here that relationships between $\xi_0$ and $\xi_1$, and g were found to be the same in reference (4) for the unsteady convection as the present relationship for the unsteady condensation.

## CONCLUDING REMARKS

The heat transfer associated with unsteady laminar film condensation was analyzed for a vertical plate. Time-dependent variation of either the uniform

wall temperature or the g-force field was considered. From the theoretical treatment, the first-order deviation of heat transfer from the instantaneous steady-state value was obtained in terms of a set of nondimensional variables $\left\{\xi_n\right\}$ and parameters Pr and $c_p(T_s - T_w)/h^o$. From the results obtained, it is possible to determine when the heat transfer with either time-dependent wall temperature or g can be computed with sufficient accuracy from quasisteady relations.

The effect of unsteady $T_w$ on heat transfer was found to increase as the Prandtl number $1/(T_s - T_w)$ and $c_p/h^o$ are increased. The unsteady -$T_w$, however, was found to have negligible effect on heat transfer when $Pr << 1$.

Unsteady -g was found to affect the condensation heat transfer substantially for the entire Prandtl number range ($0.001 \leq Pr \leq 100$) considered. It was found that the effect of unsteady g is insensitive to the Prandtl number at low Prandtl numbers. The effect, however, was found to steadily increase with the Prandtl number at high Prandtl numbers. The unsteady -g effect on heat transfer was also found to increase as $c_p(T_s - T_w)/h^o$ and g are decreased. Therefore, it is important to keep the unsteady -g effect in mind when performing a reduced -g experiment.

Finally, even though the present study was concerned only with the vertical plate, it can be directly applied to a horizontal circular cylinder by redefining the variables $\eta$ and $\left\{\xi_n\right\}$ in a manner analogous to that done in reference (4) for the unsteady free convection problem.

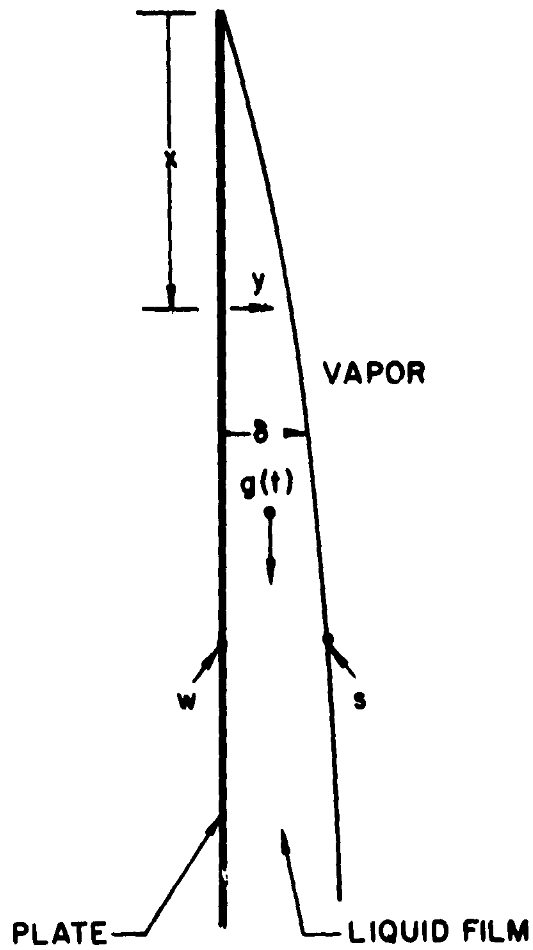


Fig. 1. Physical Model Considered.

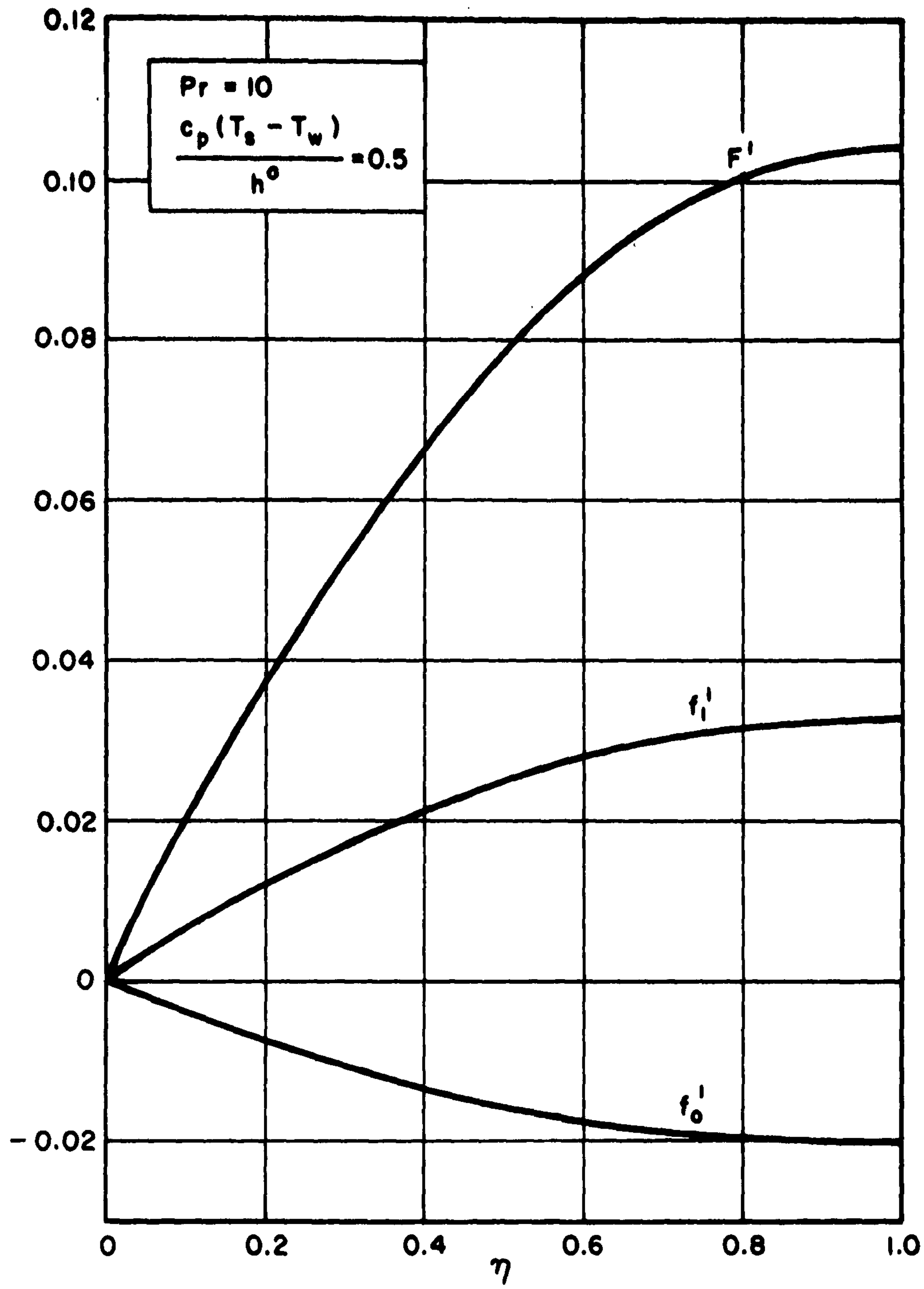


Fig. 2. Typical Nondimensional Streamwise Velocity Functions for Unsteady $T_w$.

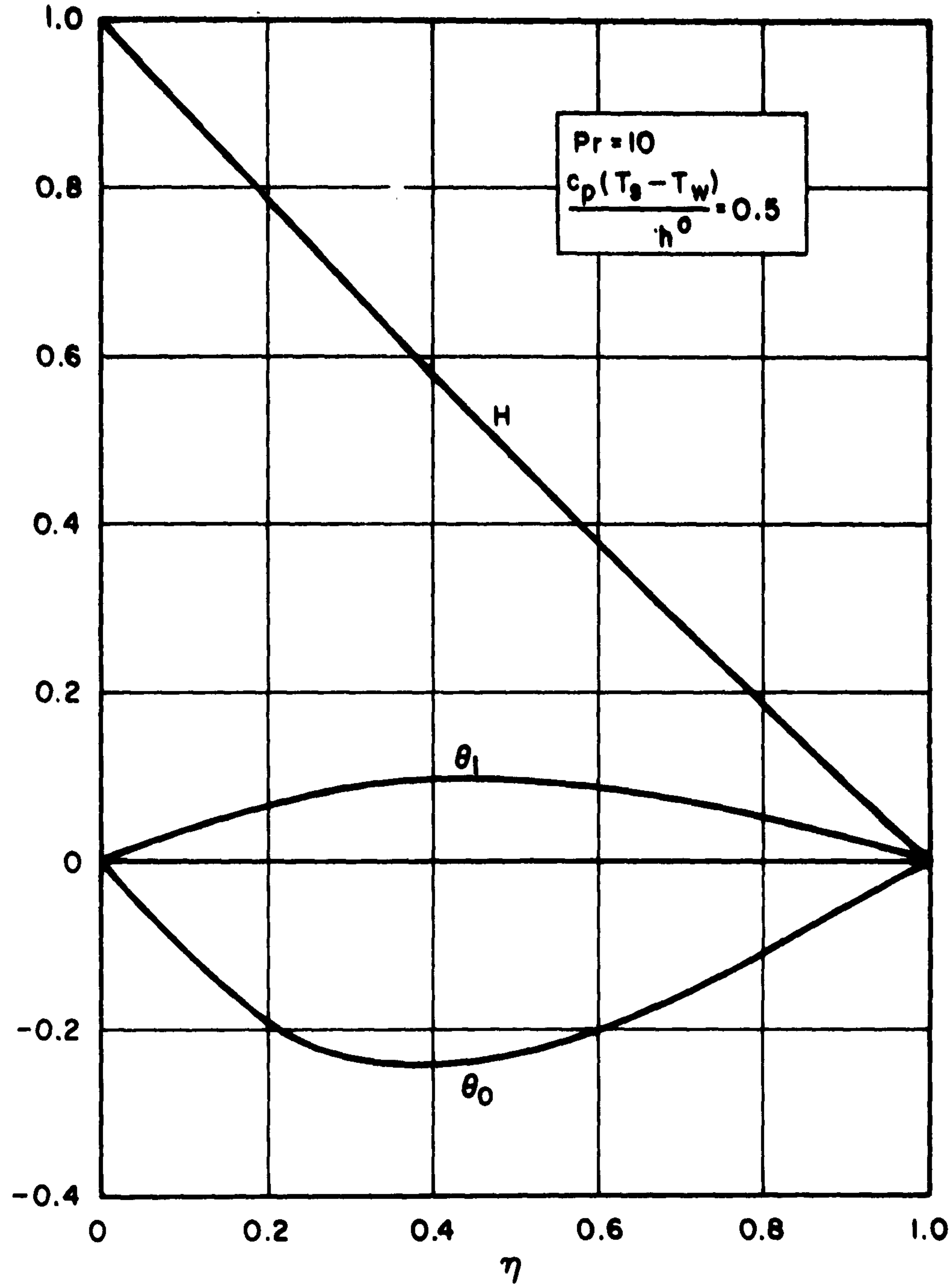


Fig. 3. Typical Nondimensional Temperature Functions for Unsteady $T_w$.

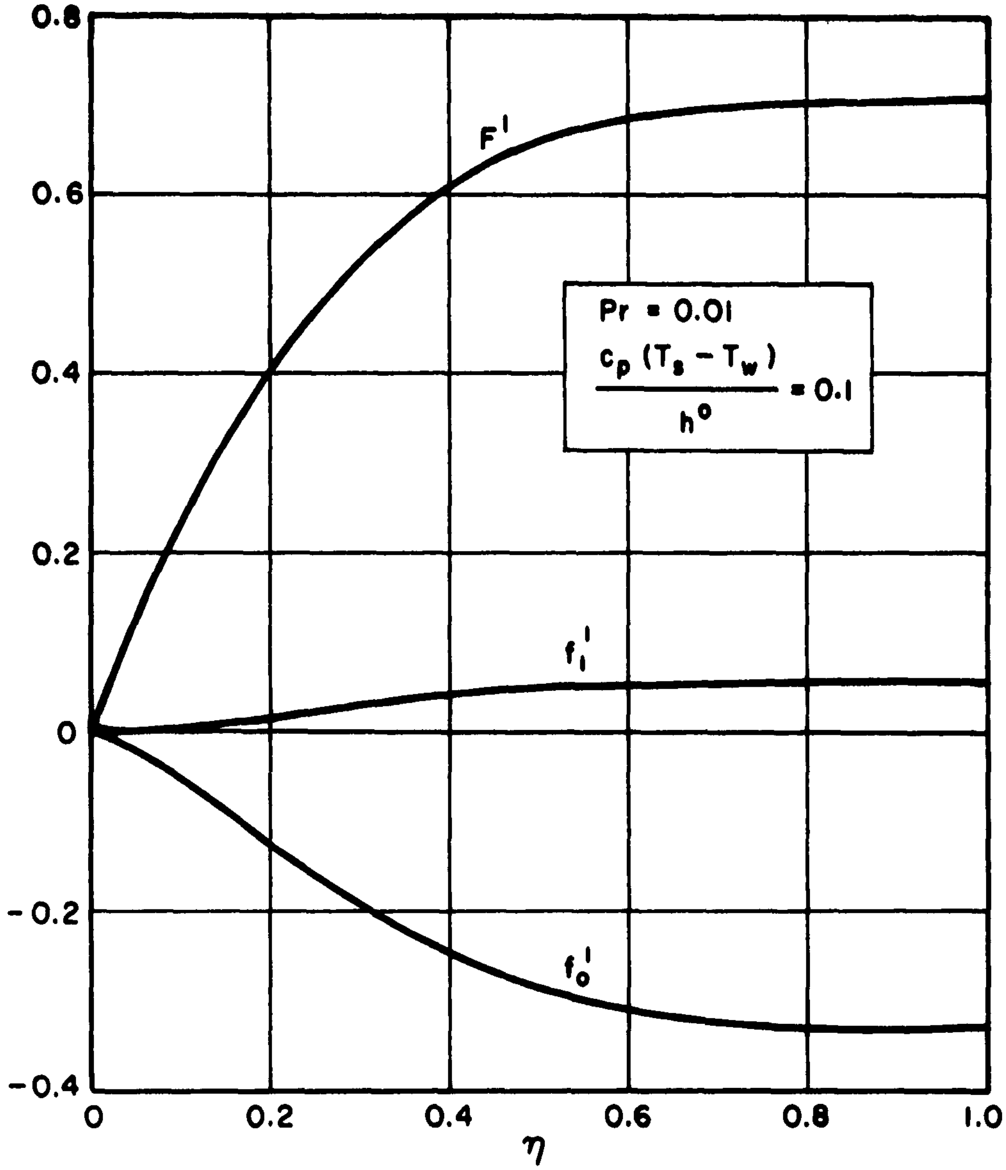


Fig. 4. Typical Nondimensional Streamwise Velocity Functions for Unsteady g.

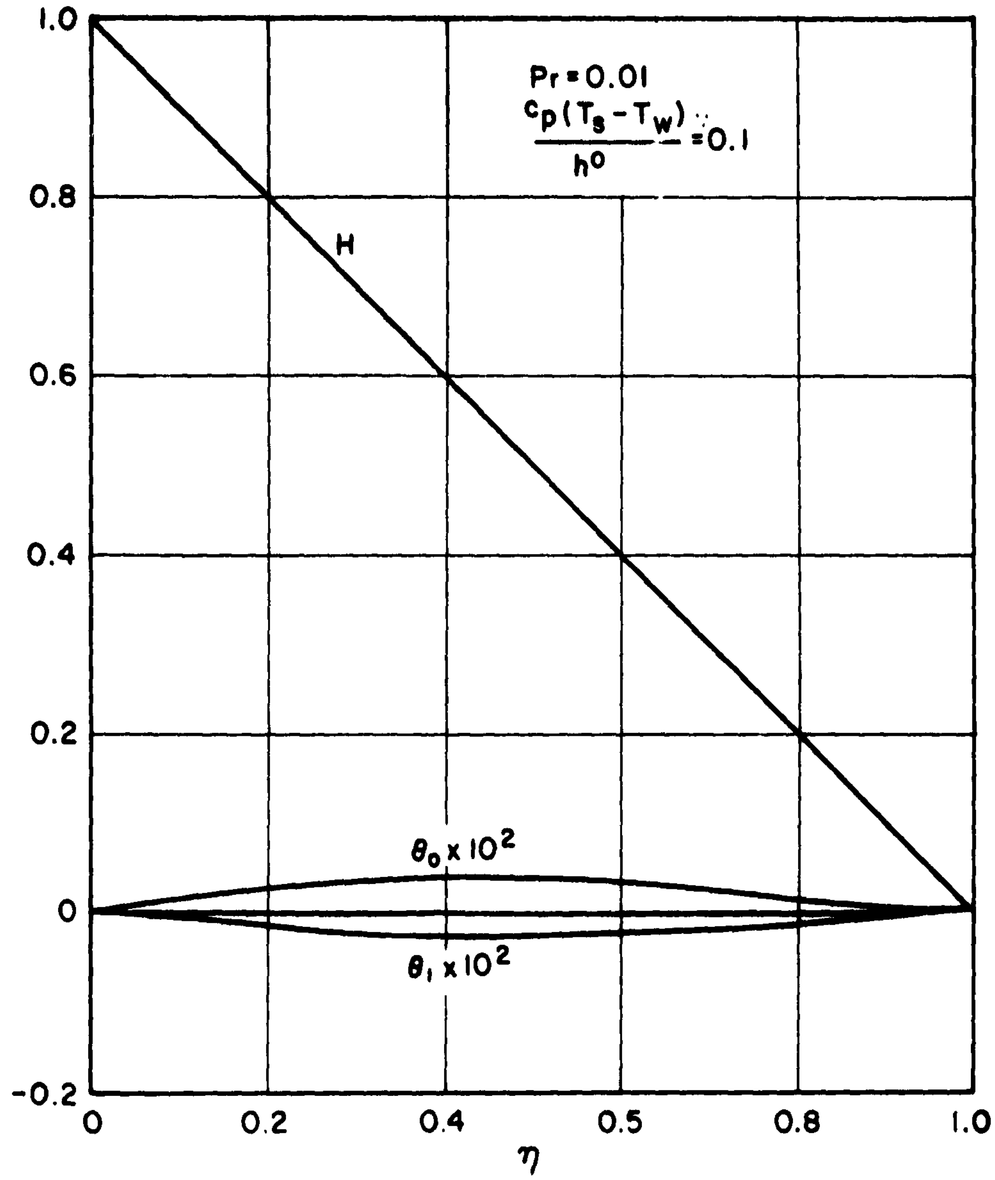


**Fig. 5.** Typical Nondimensional Temperature Functions for Unsteady g.

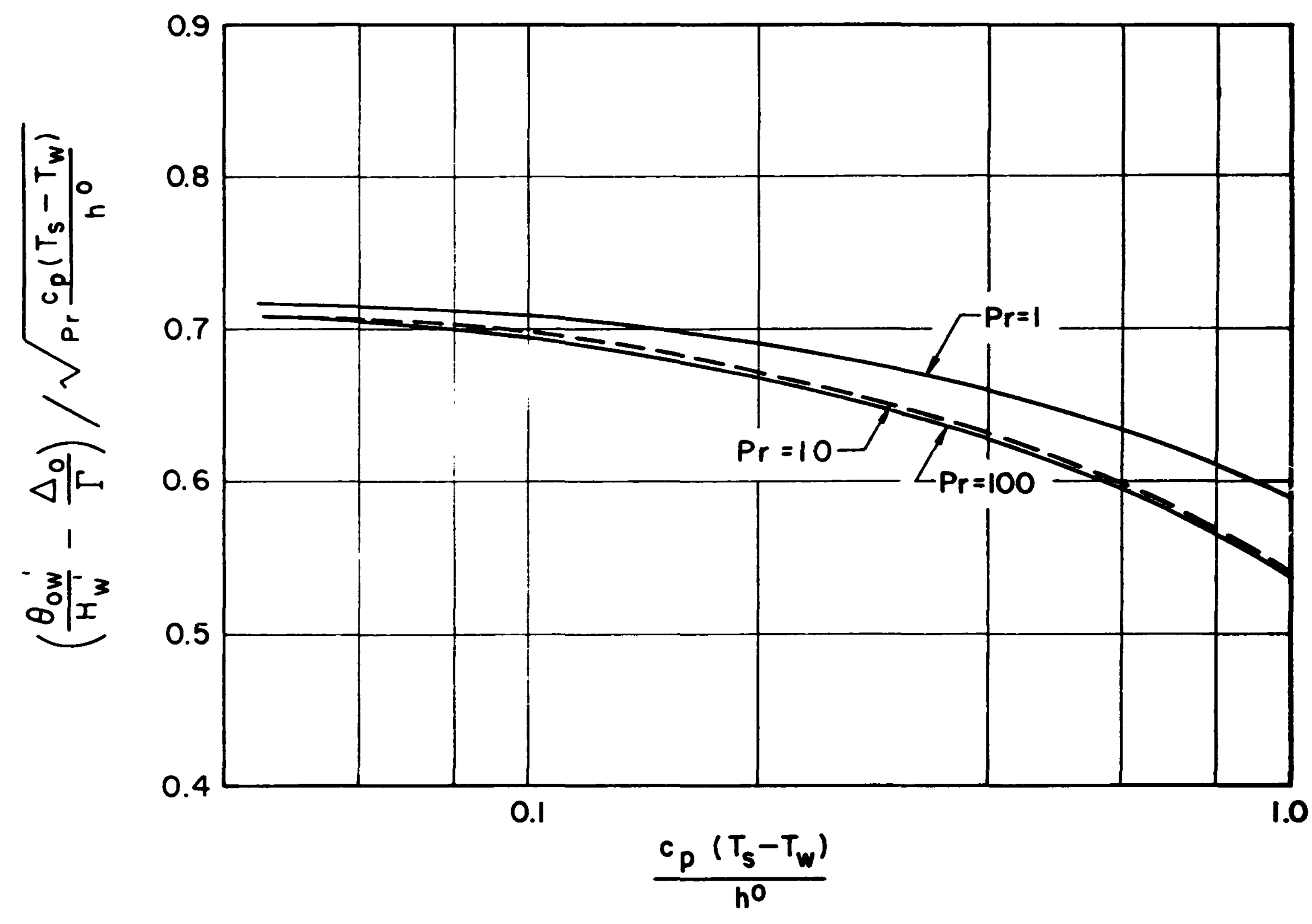


Fig. 6. Coefficient of $\xi_0$ in Equation (44) for Unsteady $T_w$.

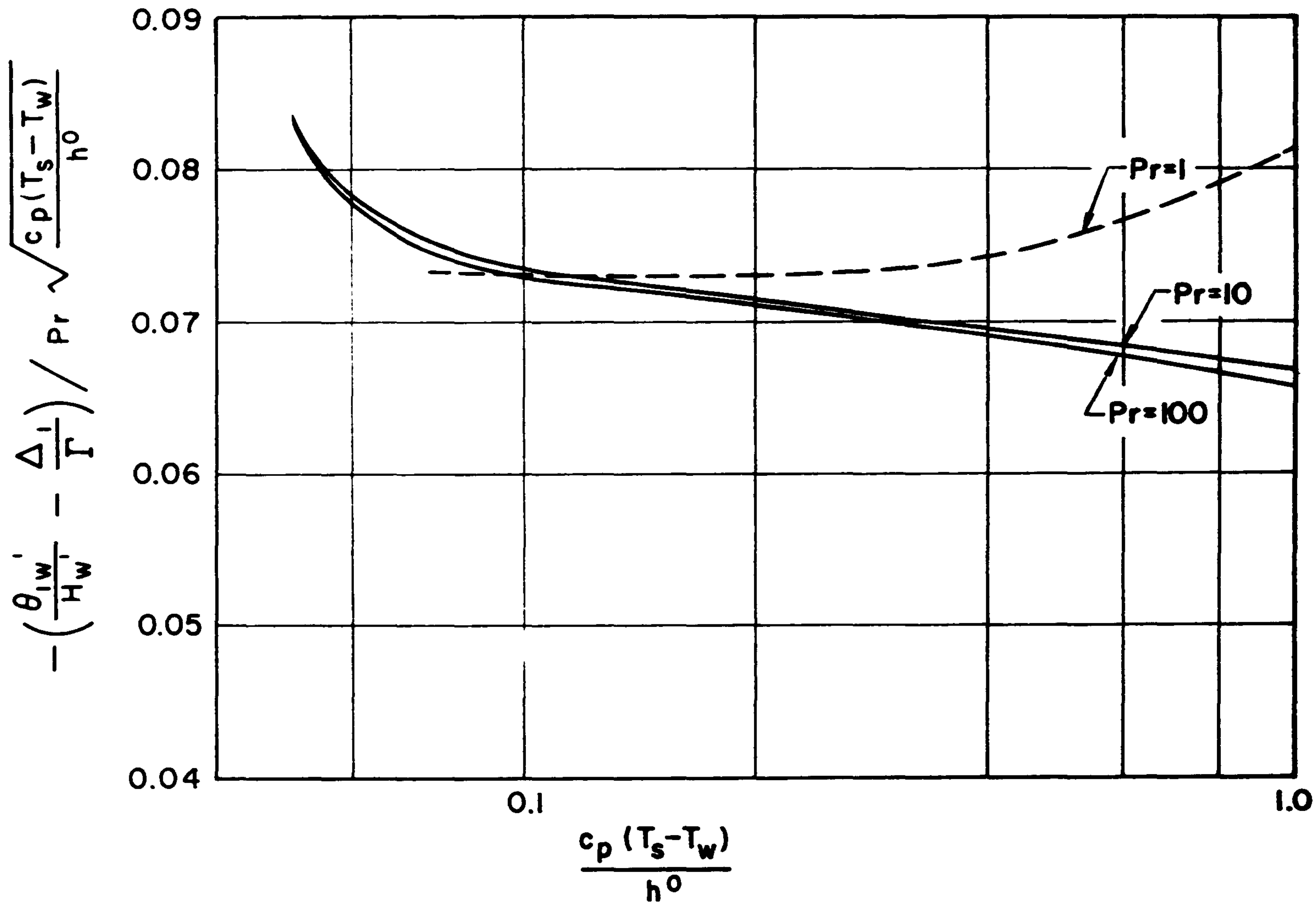


Fig. 7. Coefficient of $\xi_1$ in Equation (44) for Unsteady $T_w$.

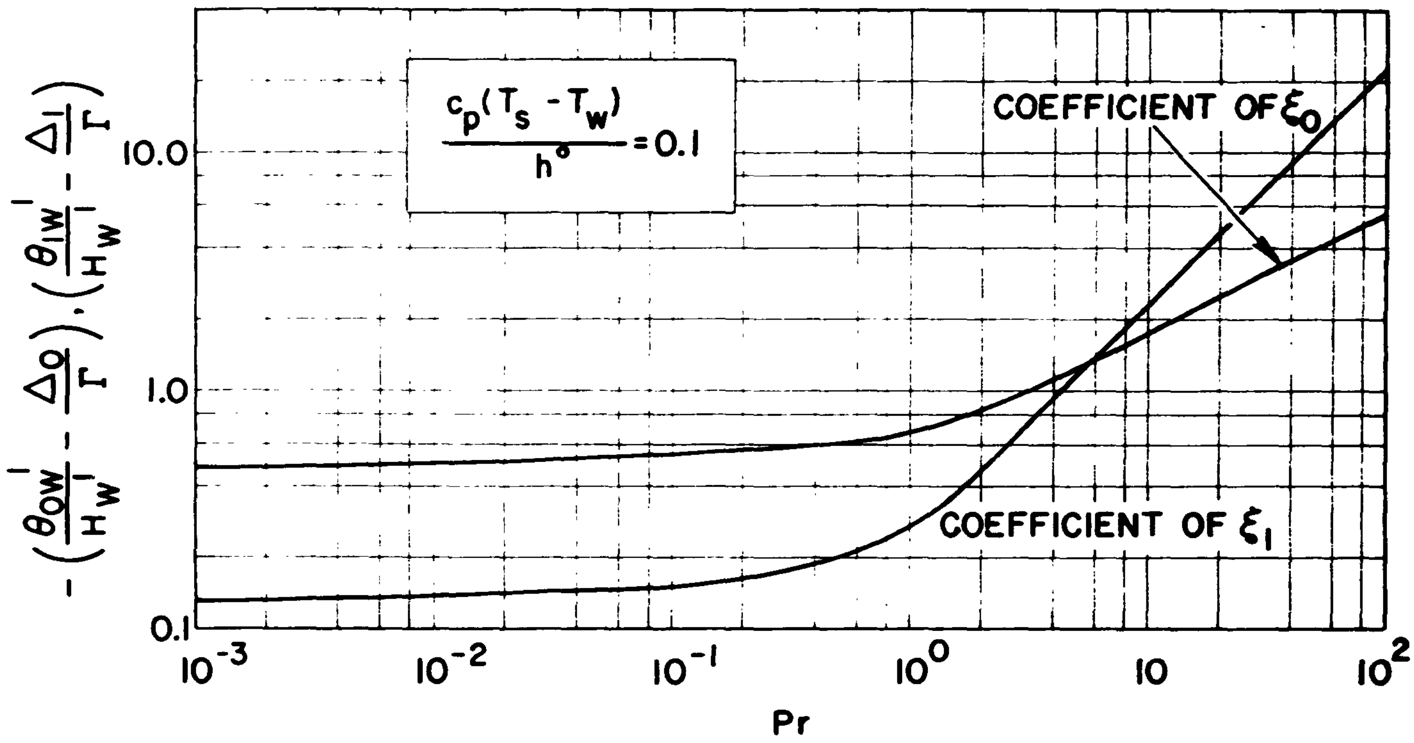


Fig. 8. Variation of Coefficients of $\xi_0$ and $\xi_1$ in Equation (44) with Respect to Pr for Unsteady g.

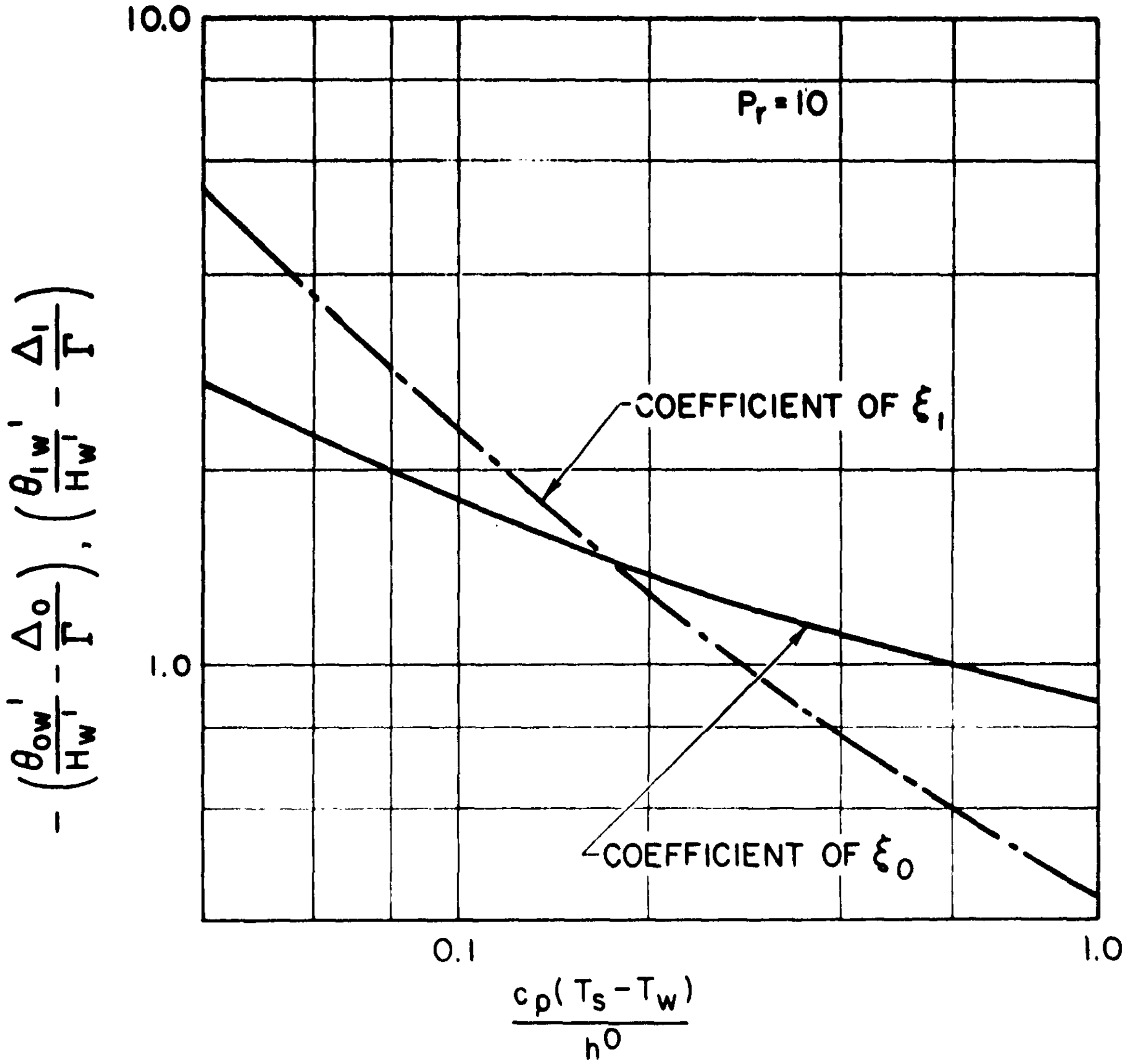


Fig. 9 Variation of Coefficients of $\xi_0$ and $\xi_1$ in Equation (44) with Respect to $c_p(T_s - T_w)/h^0$ for Unsteady g

**Table 1**

**Universal Functions for Heat Transfer (Unsteady $T_w$)**

| Pr | $\frac{c_p(T_s - T_w)}{h^*}$ | $\Gamma$ | $\Delta_0$ | $\Delta_1$ | $H_w'$ | $\theta_{ow}'$ | $\theta_{1w}'$ |
|---|---|---|---|---|---|---|---|
| 1 | 0.05 | 0.4726 | -0.0059 | | -1.005 | -0.1478 | |
| | 0.1 | 0.5616 | -0.0098 | 0.0079 | -1.010 | -0.2074 | 0.0092 |
| | 0.5 | 0.8351 | -0.0313 | 0.0138 | -1.044 | -0.4376 | 0.0385 |
| | 1.0 | 0.9858 | -0.0489 | 0.0204 | -1.081 | -0.5868 | 0.0655 |
| 10 | 0.05 | 0.2651 | -0.0102 | 0.0374 | -1.005 | -0.4652 | 0.0464 |
| | 0.1 | 0.3140 | -0.0167 | 0.0449 | -1.010 | -0.6494 | 0.0896 |
| | 0.5 | 0.4597 | -0.0463 | 0.0695 | -1.044 | -1.327 | 0.3507 |
| | 1.0 | 0.5341 | -0.0648 | 0.0821 | -1.079 | -1.716 | 0.5532 |
| 100 | 0.05 | 0.1490 | -0.0180 | 0.2104 | -1.005 | -1.470 | 0.4635 |
| | 0.1 | 0.1766 | -0.0295 | 0.2529 | -1.010 | -2.052 | 0.8938 |
| | 0.5 | 0.2579 | -0.0806 | 0.3907 | -1.044 | -4.178 | 3.473 |
| | 1.0 | 0.2992 | -0.1110 | 0.4574 | -1.079 | -5.384 | 5.438 |

Table 2

Universal Functions for Heat Transfer (Unsteady g)

| Pr | $\frac{c_p(T_s - T_w)}{h^*}$ | $\Gamma$ | $\Lambda_0$ | $\Lambda_1$ | $H_w'$ | $\theta_{ow}'$ | $\theta_{1w}'$ |
|---|---|---|---|---|---|---|---|
| 0.001 | 0.1 | 7.019 | 3.349 | -0.9035 | -1.014 | 0.0005 | -0.0003 |
| 0.01 | 0.1 | 2.412 | 1.177 | -0.3278 | -1.011 | 0.0017 | -0.0009 |
| 1.0 | 0.1 | 0.5616 | 0.3620 | -0.1421 | -1.010 | 0.0205 | -0.0139 |
| 10 | 0.05 | 0.2651 | 0.6374 | -1.133 | -1.005 | 0.0466 | -0.1148 |
| | 0.1 | 0.3140 | 0.5447 | -0.6891 | -1.010 | 0.0651 | -0.1183 |
| | 0.5 | 0.4597 | 0.4065 | -0.2425 | -1.044 | 0.1340 | -0.1404 |
| | 1.0 | 0.5341 | 0.3792 | -0.1713 | -1.079 | 0.1748 | -0.1592 |
| 100 | 0.1 | 0.1766 | 0.9507 | -3.828 | -1.010 | 0.2059 | -1.162 |

## REFERENCES


1. "Die Oberflachenkondensation des Wasserdampfes," W. Nusselt. Zeitschrift des Vereines Deutscher Ingenieure, Vol. 60 (1916), pp. 541, 569.

2. "Transient Film Condensation," E. M. Sparrow and R. Siegel. Journal of Applied Mechanics, Transactions of the ASME Series E, Vol. 26 (March 1959).

3. "An Integral Treatment of Two-Phase Boundary Layer in Film Condensation," J. C. Y. Koh. ASME Paper No. 60-WA-253.

4. "Unsteady Laminar Free Convection" P. M. Chung and A. D. Anderson. Journal of Heat Transfer, Transactions of the ASME Series C, Vol. 83 (November 1961), pp. 473-478.

5. "A Boundary-Layer Treatment of Laminar-Film Condensation," E. M. Sparrow and J. L. Gregg. Journal of Heat Transfer, Transactions of the ASME Series C, Vol. 81 (February 1959), pp. 13-18.

6. "Laminar Condensation Heat Transfer on a Horizontal Cylinder," E. M. Sparrow and J. L. Gregg. Journal of Heat Transfer, Transactions of the ASME Series C, Vol. 81 (November 1959), pp. 291-294.

7. "Film Condensation with and without Body Force in Boundary-Layer Flow of Vapor over a Flat Plate," P. M. Chung. NASA Report TN D-790 (1961).